AN INTRODUCTION TO THE STUDY OF MAGNETIC AMPLIFIERS

by

John Petersen Gordon

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I. INTRODUCTION

Definitions and History

The magnetic amplifier is a device using saturable reactors either alone or in combination with other circuit elements to secure amplification.[1] The basic element of the magnetic amplifier, the saturable reactor, is an electromagnetic device employing one or more nonlinear magnetic cores. This combination is used in alternating current circuitry to secure amplification or control by influencing the core nonlinearity.[2]

The operation of a saturable reactor may be described briefly on the basis of two windings upon a ferromagnetic core. One winding, to which d-c is applied, is called the control winding. The other is connected into an a-c load circuit, and is known as the gate winding. The d-c applied to the control winding determines the degree of core saturation. The differential permeability of the core material is dependent upon the degree of saturation, and since the winding impedance is a function of the differential permeability, it may be said that the d-c applied to the control winding influences the flow of power in the a-c load circuit.

Many circuit arrangements are possible using the basic element just described. Push-pull and polyphase connections are common, while cascaded stages are frequently used to secure higher levels of amplification. In addition, particular characteristics in the behavior of the device may be obtained through the introduction of positive and negative feedback. Notable among these characteristics is the bistable mode characterized by "on-off" operation.

The magnetic amplifier has been derived from a saturable reactor first described by C. F. Burgess and B. Frankenfeld in 1901. In a patent issued to them in 1903 was described a method for regulating electric current by means of a variable inductance reactor.[3]

It was not until 1916, despite some patent activity, that much importance was attached to the potentialities of the device. In that year E. F. W. Alexanderson used a saturable reactor in conjunction with a microphone to control the output of a 2-kw high frequency alternator.[4]

During the period between World War I and World War II a considerable number of patents were issued for basic saturable reactor circuits and elementary magnetic amplifiers. Despite this activity, very little was done in this country in the application of the device to commercial use. In Europe, particularly Germany, the potential of the magnetic amplifier was quickly recognized. Because of the extreme reliability of the device, much of the German military equipment used the magnetic amplifier.

Wartime research projects yielded much information about magnetic core materials. On the basis of this, new core materials with narrow, rectangular hysteresis loops were developed. The developments in magnetic materials coupled with advances in metallic rectifier design resulted in a vast amount of progress in the field of magnetic amplifiers in the years immediately after World War II.

At the present time a great deal of work is being done in the outlining of the theory of magnetic amplifier operation. Up to this point the operation of the magnetic amplifier has been described almost entirely in empirical terms.

The magnetic amplifier has found its way into a vast number of commercial applications. The reason for this activity lies in the many characteristics of the magnetic amplifier which give it distinct advantage over other amplifying devices. It is a unit having extreme reliability, a long life, ruggedness, no warm-up time, high efficiency, a minimum of maintenance problems, and high temperature capabilities.[5] The main disadvantage of the device is its slow response time. Other disadvantages which may be overcome in certain situations are the cost and size of the unit.

Characteristics of Magnetic Amplifier Core Materials

The magnetic materials used in the construction of magnetic amplifiers are characterized by the abrupt change between the high and low differential permeability regions of the dynamic hysteresis loops. This abruptness lends itself to some simplifying assumptions that are adopted in many treatments of magnetic amplifier theory. A hysteresis loop that is typical of the materials used in magnetic amplifiers is shown in figure 1. The solid curve represents the dynamic loop for a material known commercially as Orthonol.

The dotted portions of the curve in figure 1 depict the minor hysteresis loops. These loops occur when an alternating magnetic

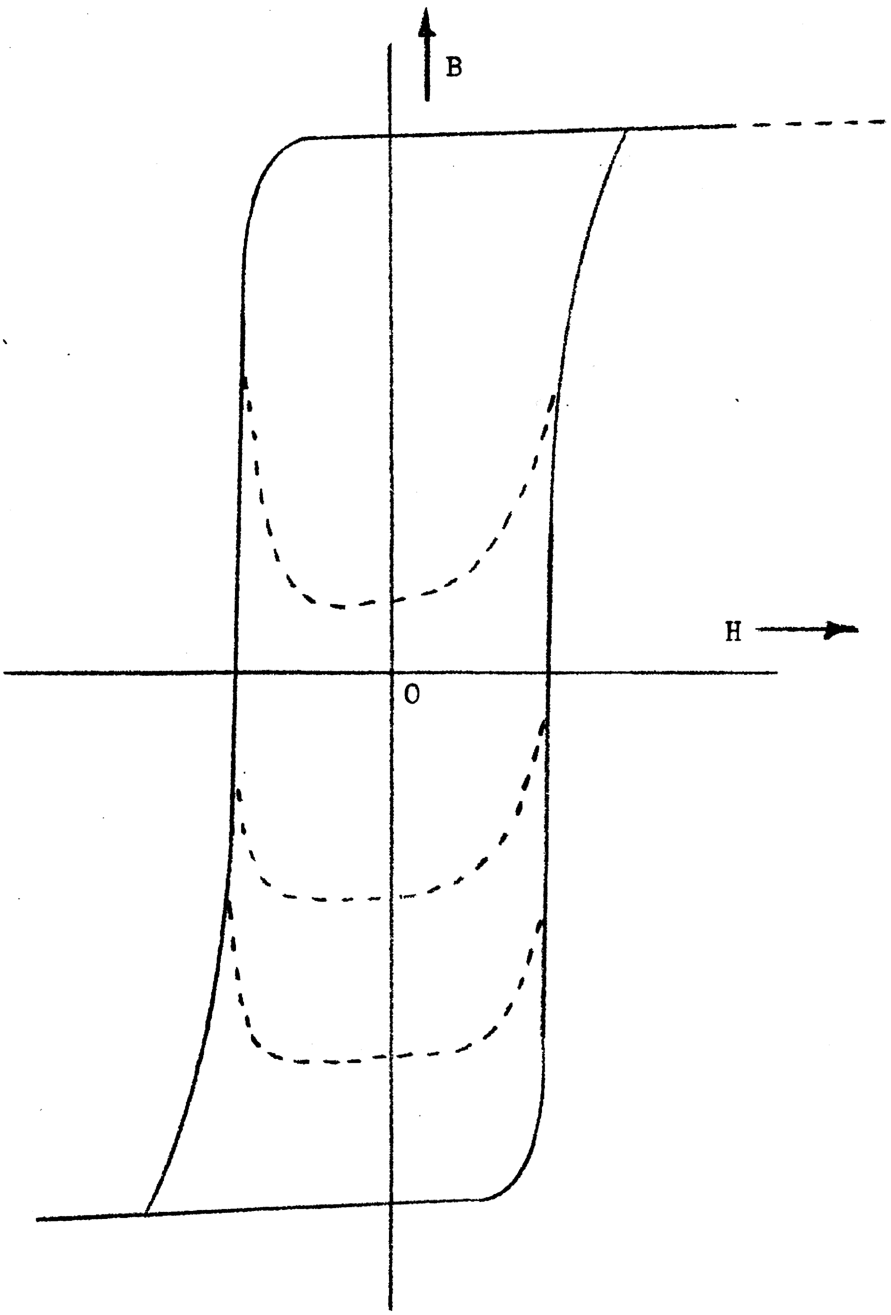

Figure 1 A typical dynamic hysteresis loop for Orthonol.

field is superimposed upon a constant magnetic field and applied to a ferromagnetic core.[6] It will be seen later that the minor loops of the hysteresis curve become an integral part of the theory of saturable reactors and magnetic amplifiers.

As seen in figure 1, the analysis of saturable reactor and magnetic amplifier operation will be difficult because of the non-linearity of the hysteresis loop. This difficulty can be largely overcome by considering a hysteresis loop of zero area, whose high and low differential permeability regions are drawn as straight lines. The simplified hysteresis loop that will be used in this presentation is shown in figure 2. The fact that the loop has zero area implies that the power loss in the core due to hysteresis and eddy currents is low. The high slope and zero slope regions simplify the non-linear relationships that would be present in the analysis. It must be remembered, though, that while these linearizations prove useful in various aspects of magnetic amplifier analysis, one must not lose the appreciation for the extreme non-linearity that actually exists.

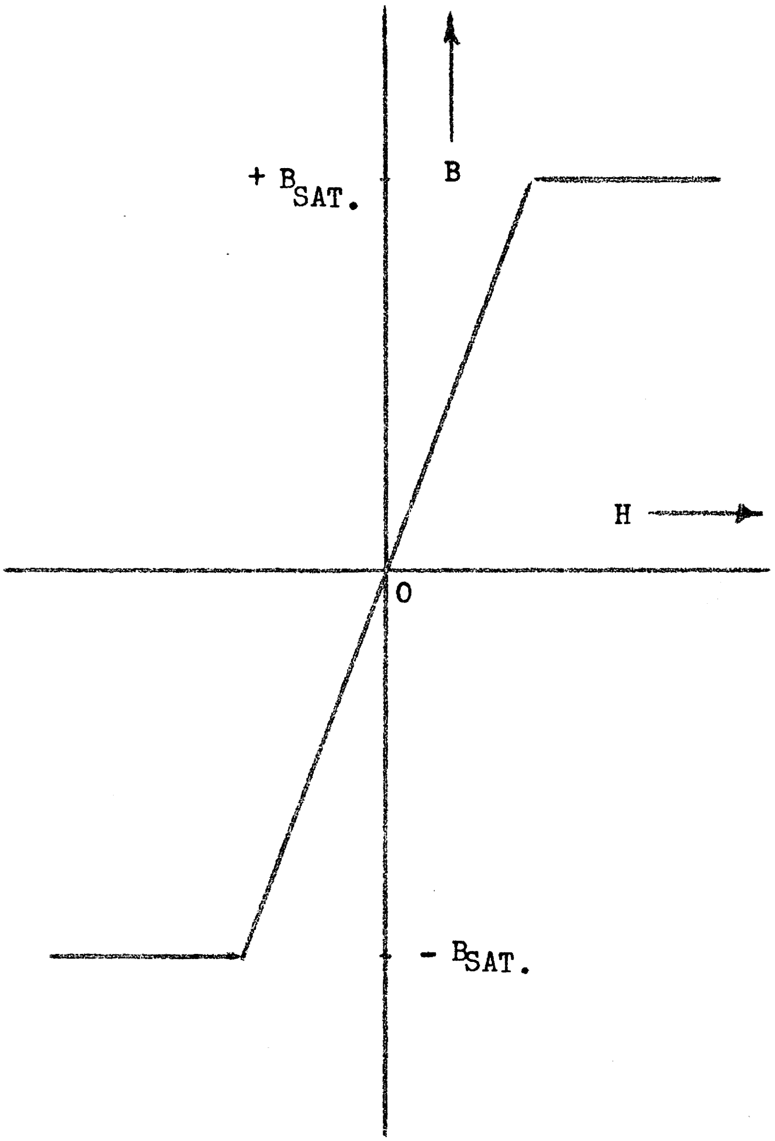

Figure 2 An idealized hysteresis loop for use in saturable reactor and magnetic amplifier analysis.

II. THE BASIC SATURABLE REACTOR

Voltage and Flux Relations

An understanding of the basic voltage and flux relationships is necessary in the study of saturable reactor operation. To establish these relations, consider a circuit such as that shown in figure 3. A gate winding with N_G turns is wound on one leg, and a control winding with N_C turns is wound on the other leg of a core made of arbitrary material. The application of Kirchhoff's voltage law to the gate circuit with the control circuit open yields the equation

$$e_P = i_L R_L + N_G \, d\phi/dt \qquad 1$$

Equation 1 will have particular significance with respect to saturable reactor theory when it is integrated between the limits of t_1 and t_2. The resulting equation 2 describes the volt-second, or flux linkage relationship in the circuit.

$$\int_{t_1}^{t_2} e_P \, dt = \int_{t_1}^{t_2} i_L R_L \, dt + N_G(\phi_2 - \phi_1) \qquad 2$$

Equation 2 indicates that the area under the volt-second curve for the load resistance plus the area under the volt-second or flux linkage curve of the gate voltage must be equal to the area under the source volt-second curve. The area under the flux linkage curve for the gate winding is limited by allowable range of core flux. The variation in flux as dictated by the hysteresis loop is limited to $+\Phi_s$ to $-\Phi_s$; the positive and negative values of saturation

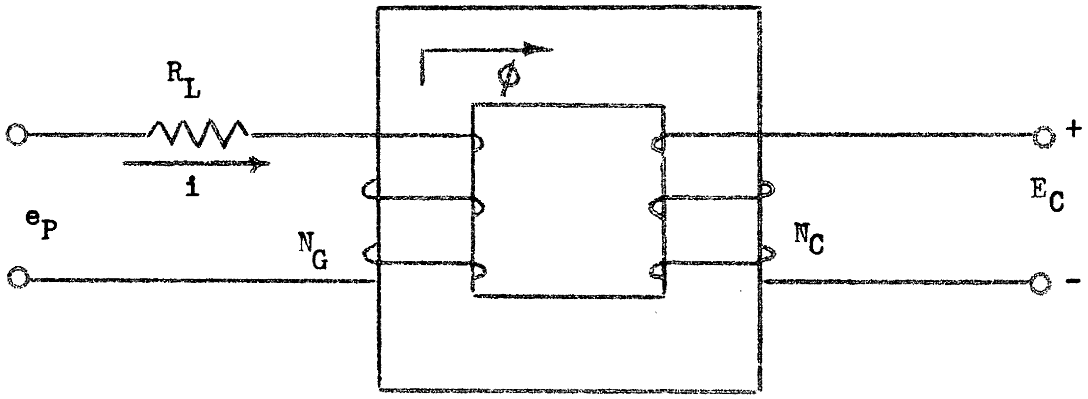

Figure 3 Basic saturable reactor circuit.

flux. The applied voltage which will yield the maximum range of flux variation is called the saturation voltage, an important reactor characteristic. For applied voltages less than the saturation voltage virtually all the voltage will be developed across the gate winding with an almost negligible voltage developed across the resistor (due to exciting current). Should the applied voltage be larger than the saturation voltage equation 2 indicates that the core will be saturated during the interval, and the applied voltage will be developed across the resistor during this saturation interval. The presence of voltage across the load resistance R_L, indicates the existence of gate or load current.

Simple Saturable Reactor Operation with Open Control Circuit

When a sinusoidal voltage is applied to the gate circuit of figure 3 with the control circuit open the resulting operation depends upon the magnitude of the applied voltage. Should the voltage applied be less than the saturation voltage then according to equation 2 the core will not saturate. The waveforms of flux and voltage over a full cycle of the applied voltage are shown in figure 4 as the dotted wave forms. Since the core does not saturate the only voltage developed across the resistor may be attributed to exciting current.

On the other hand, a voltage larger than the saturation voltage will cause the core to saturate. Notice in the solid curves of figure 4, that when the flux reaches saturation, the voltage that was developed across the gate winding now appears across the resistance.

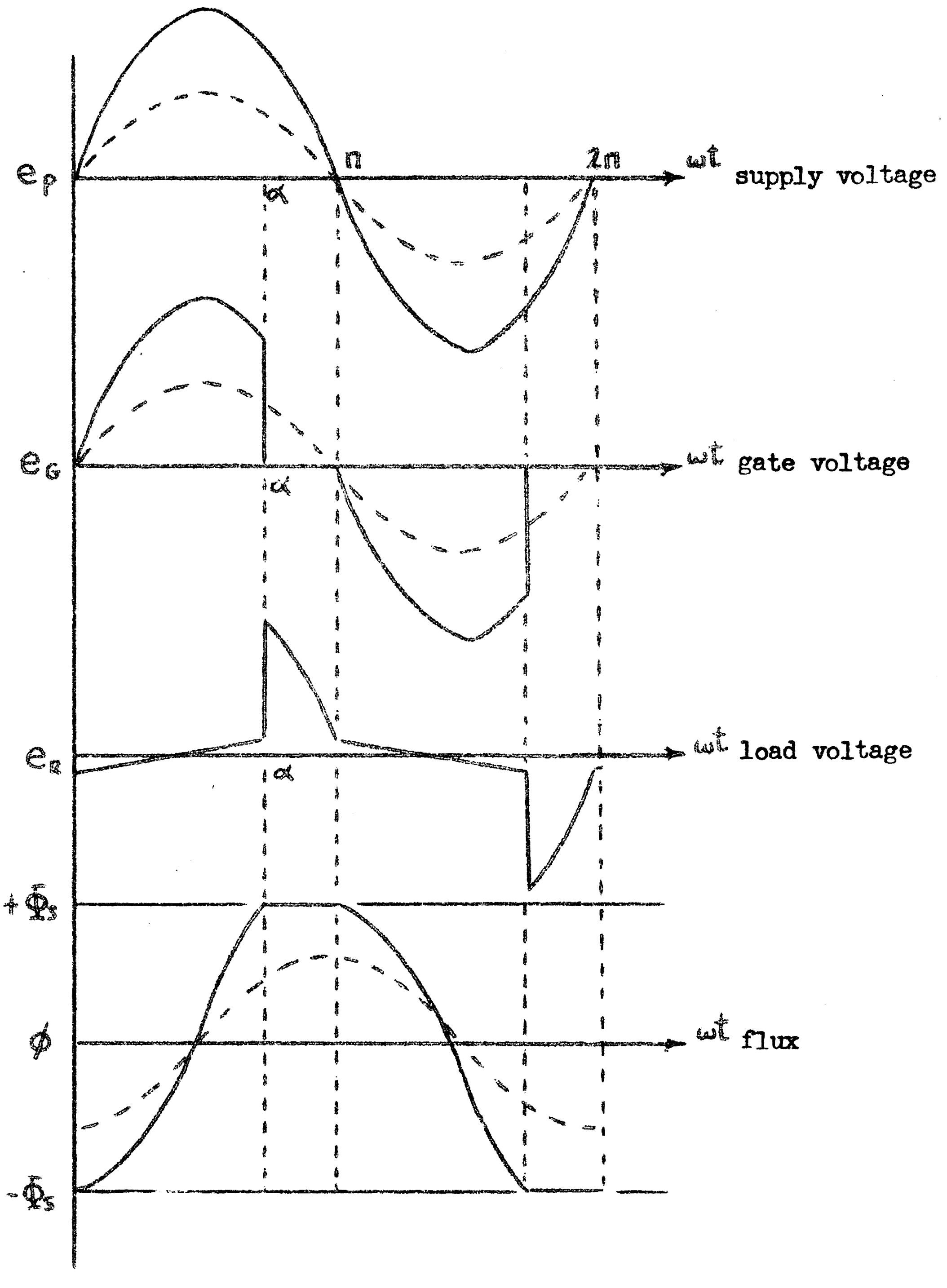

Figure 4 Voltage and flux waveforms in a basic saturable reactor circuit with zero control current.

This is in complete agreement with equation 2. The interval when the core is saturated is referred to as the saturation interval, and is characterized by the existence of current in the gate circuit. In figure 4 this is indicated by the voltage across the resistance. The point at which the core saturates, initiating the gate current waveform, is frequently referred to as the firing angle, α . This is due to the similarity to the firing of a thyratron.

Simple Saturable Reactor Operations with Control Voltage Applied

The application of a d-c control voltage results in a flux variation that follows one of the minor loops such as those shown in figure 1. With an a-c voltage applied equal to the saturation voltage the core is made to saturate as a result of the d-c field. The flux waveform, indicating saturation of the core, is shown in figure 5. Again, while the core is saturated current exists in the gate circuit. During the second half cycle, at angle $(\alpha + \pi)$ radians; while the core does not saturate, there is no appreciable change in flux. Hence the voltage will appear across the resistor as before. From this it is seen that the average value of the gate current depends upon the magnitude of the control current. The control current sets the level of the flux variation.

The reactor circuits discussed on the preceding pages were introduced as simple examples of flux and voltage behavior in circuits including rectangular hysteresis loop core materials. Reactors of this form present serious disadvantages in commercial usage due to

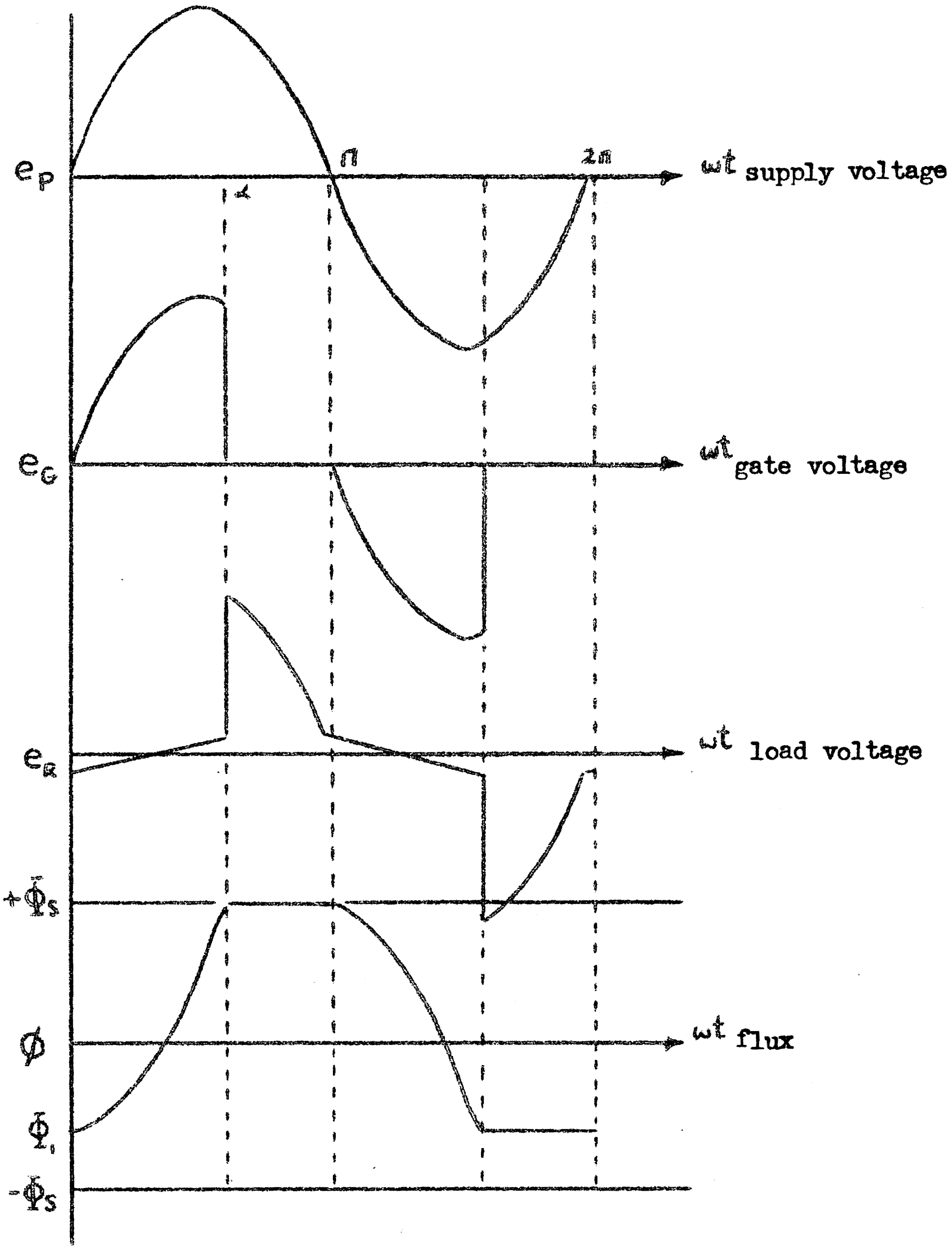

Figure 5 Voltage and flux waveforms in a basic saturable reactor circuit with applied control voltage.

the mutual induction between gate and control windings. In its practical form the reactor utilizes two core assemblies with control windings connected so as to eliminate any fundamental induced voltages in the control circuit.

III. THE SERIES CONNECTED SATURABLE REACTOR

The form in which the more practical saturable reactor appears is varied. As already mentioned, the circuit may consist of two complete core assemblies as shown in figure 6. In addition the circuit may be found in conjunction with a three-legged core. The gate windings wound on the outside legs, and the control winding on the center leg. The windings in any case are wound in such a manner so as to eliminate the undesirable mutual effect between gate and control winding. In the case of the circuit in figure 6 the gate windings are connected in the same fashion, while the control windings are connected opposite from each other. Where the three-legged core is used the gate windings are connected in opposition since there is only the one control winding. In addition to the aforementioned arrangements the gate windings may be connected in series or parallel.

In dealing with the theory of two core saturable reactors the idealized core characteristics are used. In addition the transient period is considered complete, and all waveforms depict steady state conditions. The result is an idealized steady state analysis of the operation of a saturable reactor with a simple resistance load.

Series Connected Saturable Reactor with low Control Circuit Impedance

The introduction of the factor of control circuit impedance into the discussion divides the analysis of saturable reactors into

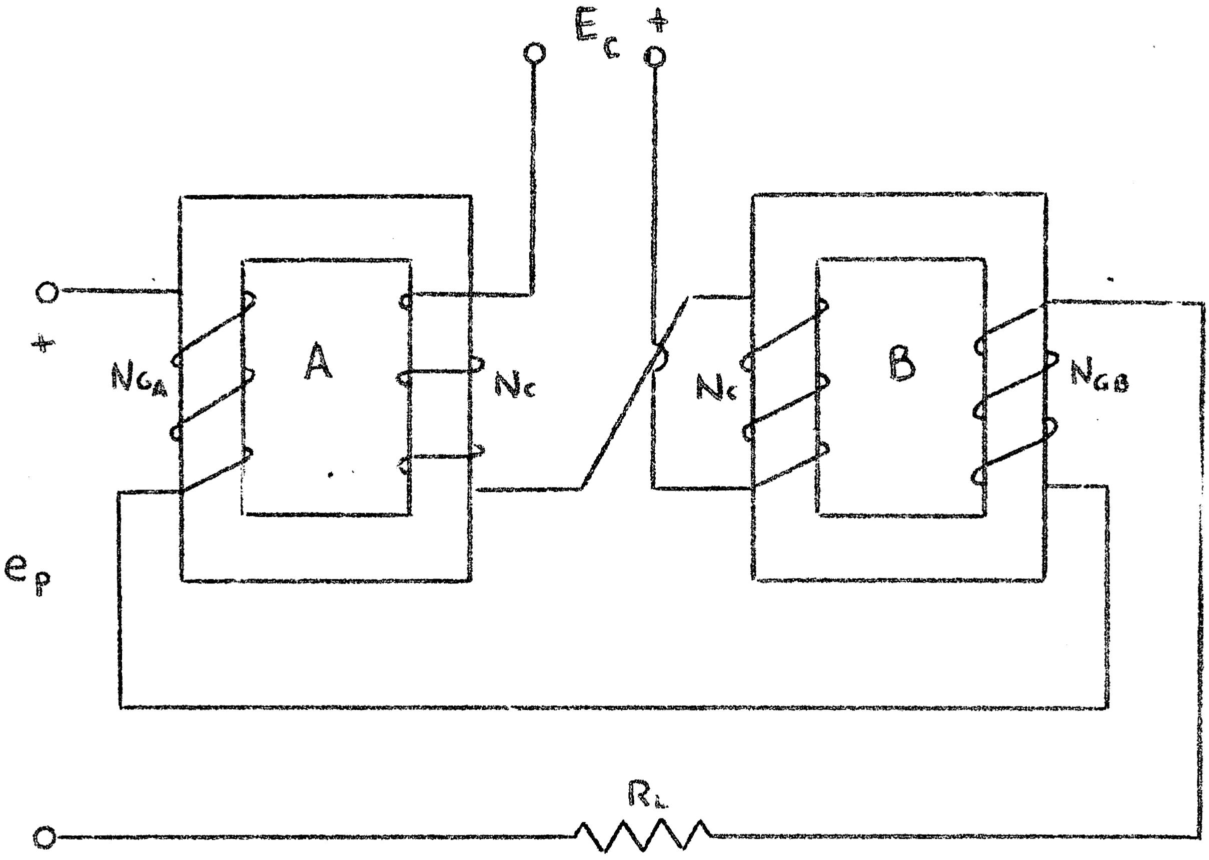

Figure 6 Series connected saturable reactor.

two categories, the first of which centers upon the case where the control circuit impedance is low. Under these conditions the circulation of free even-harmonic currents in the control circuit is unimpeded. This type of operation is referred to as natural magnetization.[7] The other category deals with the suppression of even-harmonic currents, and will be dealt with briefly at a later time.

When the control current in the circuit of figure 6 is zero the operation will depend solely upon the magnitude of the applied a-c voltage. Should this voltage be less than the saturation voltage the reactor cores will offer a high impedance in the gate circuit. The only current that will exist in the gate circuit will be the exciting current. However should the voltage applied be raised to a value greater than the saturation voltage the two cores will saturate and unsaturate during the cycle.

When the smaller value of voltage is applied the dotted waveforms in figure 7 depict the circuit conditions. The applied voltage divides equally between the identical gate windings. The flux varies in a sinusoidal manner between the saturation limits. No current except the exciting current exists in the gate circuit.

With the application of the larger value of supply voltage, it will also be divided equally across the gate windings until such time as the cores saturate. At this point, the firing point, the voltage will appear across the resistance in the gate circuit and gate or load current will exist. These conditions are shown in figure 7 as the solid curves.

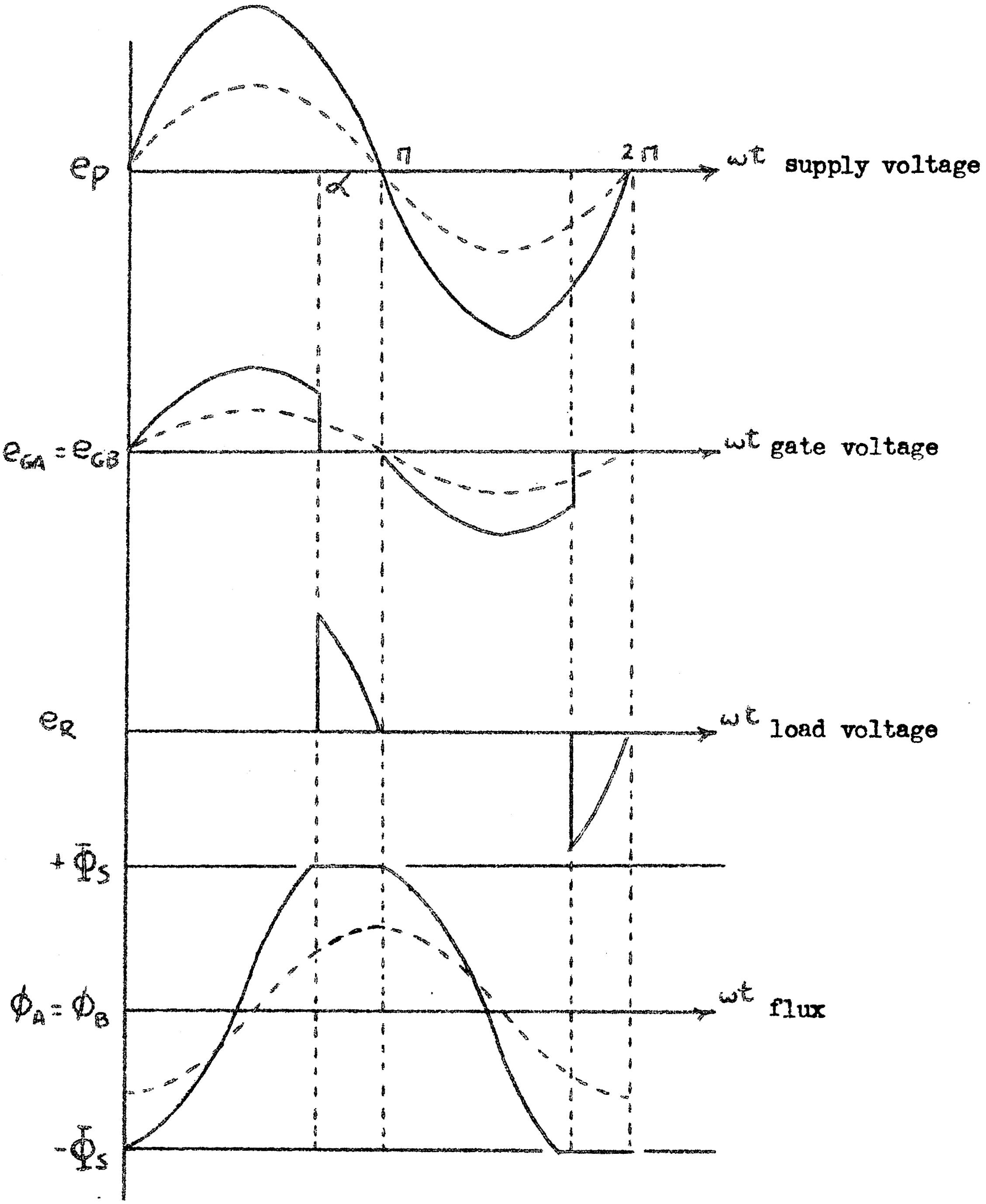

Figure 7 Series connected saturable reactor with zero control current.

With control current in the control circuit of figure 6 it is interesting to note that for polarities as shown the magnetomotive forces from gate and control windings add in one core, A; and subtract in the other core, B. The result of this is seen in the orientation of the flux waveforms in figure 8. The flux in core A, ϕ_A, is biased upward to a point Φ_1 determined by the current in the control circuit. Since the control current is pulsating in nature it is the d-c component of this current that influences the flux level. The flux in core B, ϕ_B, is in negative saturation where it remains. These conditions represent the starting conditions for a single cycle of operation.

The operation of the saturable reactor under the circumstances just outlined may be divided into four distinct modes of operation.[8] The waveforms for each mode are shown in figure 8 along with an equivalent circuit for each mode representing the circuit conditions during the periods involved. The voltage applied under these conditions is equal to the saturation voltage.

In the first mode of operation the two gate windings are coupled together by the low impedance control circuit. They will present the same impedance, and the applied voltage will divide between them equally. Since neither core has reached saturation yet, the only current that exists in the load circuit is the exciting current. This mode of operation is commonly called the exciting interval.

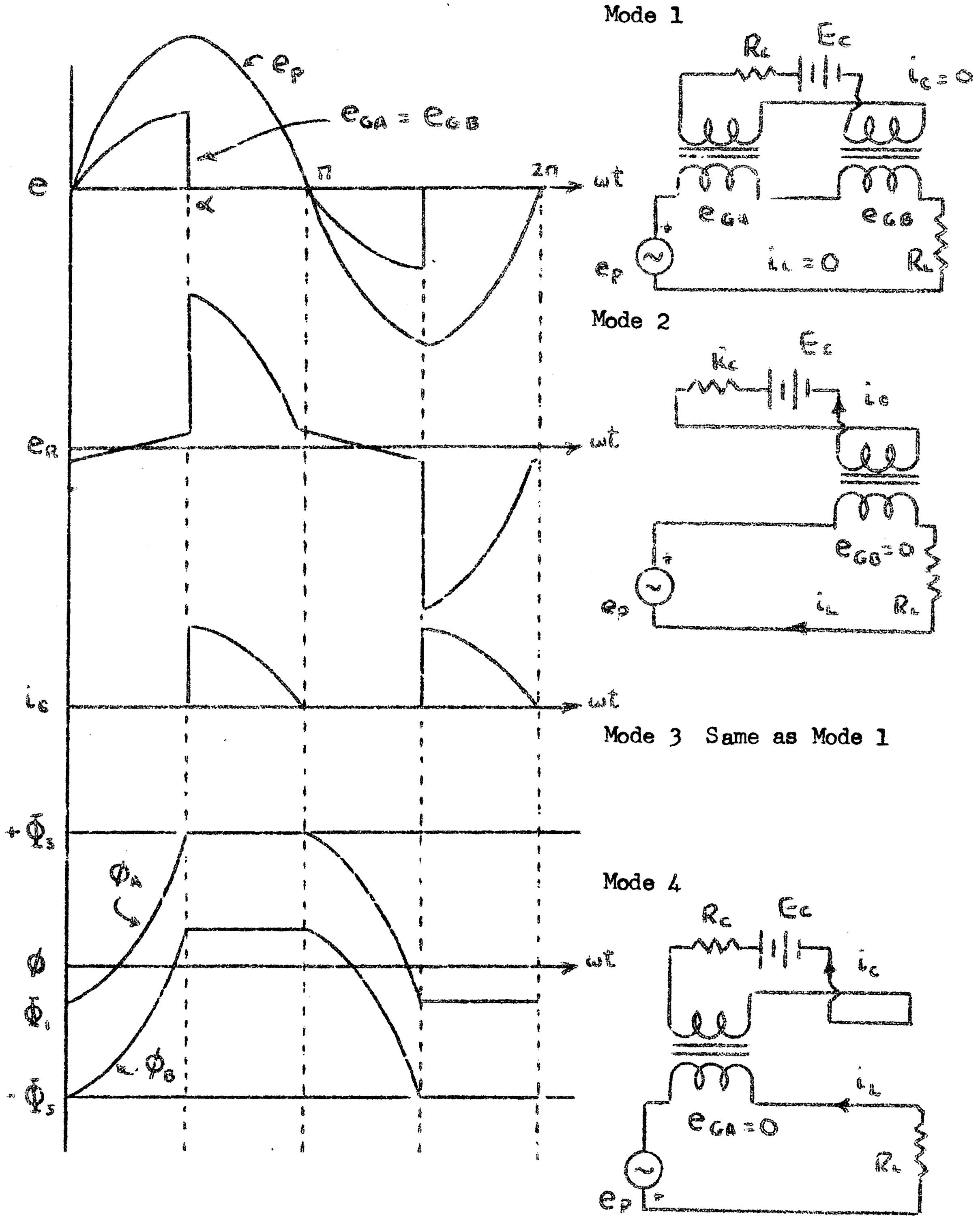

Figure 8 Series connected saturable reactor operation with control current.

As core A saturates the second mode begins. When saturated, ϕ_A will remain constant causing the voltage across gate A, e_{GA}, to become zero. At the same time, since the control circuit acts as a short circuit on the gate winding of B, e_{GB}, the voltage across gate winding B will go to zero. Since e_{GB} is zero there is no further increase in ϕ_B. With both gate voltages equal to zero the entire supply voltage will appear across the load resistance R_L and load current i_L will exist. Since core B is unsaturated its windings will act as a transformer dictating that a current in the control circuit will exist such that

$$i_c = i_L \, N_G/N_C \qquad 3$$

This mode of operation is commonly called the saturation interval.

With the beginning of the negative half cycle of the applied voltage core A will come out of saturation thus beginning the start of the third mode. Since both cores are now out of saturation, very nearly the same situation exists as it did in the first mode. The only difference being the opposite polarities. The applied voltage will be divided equally between the two gate windings. The only current existing in the load circuit will be the core exciting current.

Finally core B will be driven into saturation signalling the fourth or last mode of operation for the cycle. The entire supply voltage will appear across the load resistance indicating the existence of load current. Since core A is now acting as the current transformer, and since the relationship between gate and control winding is opposite of that for core B the current in the control circuit will again appear as a positive pulse.

Analysis of the control current waveform by the Fourier series will indicate that it is made up of a d-c component and all even order harmonics. This then is the reason for calling the operation of a saturable reactor with low control circuit impedance natural magnetization, or as some authors call it free even harmonic current operation.

It is interesting to note at this point that when Kirchhoff's voltage law is applied to the gate circuit, and the result integrated as in equations 1 and 2 that the volt-time and flux linkage relations are in complete agreement with those indicated by the waveforms.

Law of Equal Ampere Turns

During the interval in which core A is saturated, core B, while sustaining no flux change, remains unsaturated. From figure 2 it may be seen that when the core is unsaturated the only mmf is that due to exciting currents. Compared with the magnitudes of load and control currents this may be considered negligible in order to establish a relationship between load and control current. During both saturation intervals the current that exists in the load circuit must be balanced by current in the control circuit. This relationship may be shown as

$$i_L N_G + i_C N_C = 0 \qquad 4$$

Denoting I_L as the average value of the rectified load current, and I_C as the average value of the control current, the current relationship in equation 4 can be put into a more useful form. Integration of equation 4 will lead to the integrals representing the average values just defined, I_L and I_C. Equation 4 may be rewritten as

$$I_c N_c = I_L N_G \qquad 5$$

This equation is commonly referred to as the law of equal ampere-turns. Equation 5 is valid under all conditions except in cases where the saturation intervals overlap, and also where the gate winding mmf during the exciting interval is not negligible.[9]

Examination of the waveforms in figure 8 reveals that increases in control current cannot produce unlimited increases in load current. The largest value that I_L may attain occurs when the firing angle is zero. Under these conditions there is a maximum rectified average load current as described in equation 6.

$$I_{L,MAX.} = E_P/R_L \qquad 6$$

Where E_P is the average value over half a cycle of the applied a-c voltage and R_L is the load resistance as designated in figure 6.

When the control current is zero the firing angle is π radians, and the load current is zero. As the control current is increased the firing angle α, decreases; resulting in an increased average load current. The limit occurs when α is zero. These relationships will be discussed in a later section dealing with the reactor characteristic curves.

Saturable Reactor Operation Under Forced Magnetization

When the control circuit has characteristics such that even harmonic currents are suppressed in the control circuit the reactor is said to be operating under conditions of forced magnetization. This type of operation tends to produce a load current waveform which is flat-topped. The high impedance control circuit can be attained either by inserting a series resistance, or a series inductance. The series resistance effects the power loss in the control circuit. The series inductance, on the other hand, does not appreciably effect the power relation, but instead gives the control circuit a large time constant.

When the control circuit impedance is high the energy source in the control circuit may be considered as a constant current source. The fact that the control current is constant dictates an important factor in this mode of operation. From figure 6 notice that the net mmf for core A may be expressed as $i_C N_C + i_L N_G$; while the net mmf for core B may be expressed as $i_C N_C - i_L N_G$. If i_C is to remain constant it can be deduced from the two expressions for net mmf that both cores may never be unsaturated together.[10]

The waveforms of current, voltage, and flux for a saturable reactor operating under forced magnetization conditions are shown in figure 9. Drawn for a relatively low value of control current these waveforms show the flat-topped characteristic of the load current.

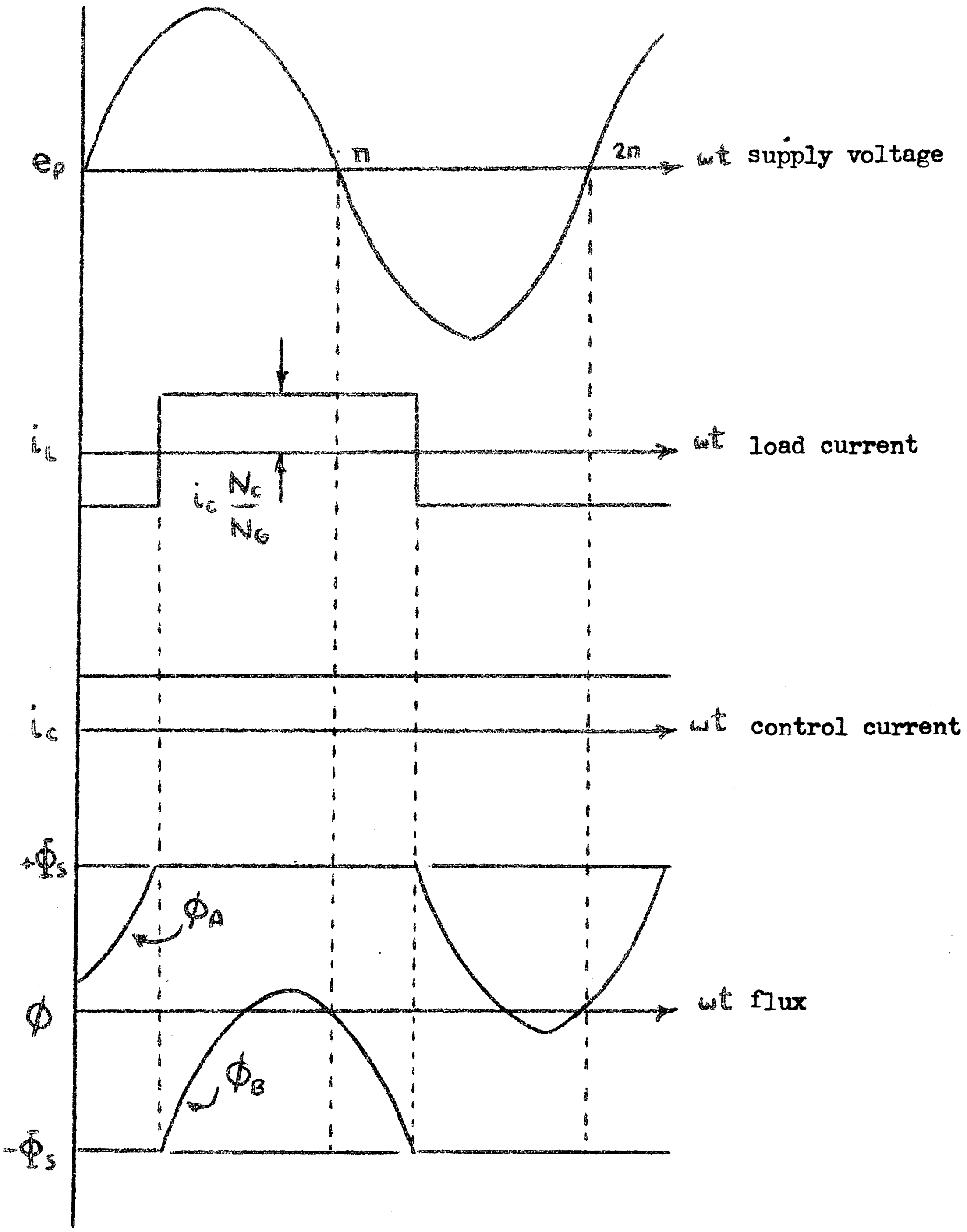

Figure 9 Series connected saturable reactor operation under forced magnetization conditions.

Since both cores may not be unsaturated at the same time, over a cycle of operation first one and then the other pair of windings will act according to the law of equal ampere turns. The net effect is that the control current is reproduced in the load circuit with a magnitude defined by the turns ratio, and a direction defined by the nature of the windings. Since the two gate windings are wound opposite in sense, with respect to the control winding, the current waveform will have the appearance of a square wave. The change from positive to negative values and vice versa occurs at the points where both cores are saturated at the same time.

The conditions of forced magnetization are not common in practical applications. One of the popular applications of this mode of operation is in the metering of large direct currents. The square wave output is rectified, and measured as d-c. The circuit is said to act as a d-c transformer.

IV. THE PARALLEL CONNECTED SATURABLE REACTOR

In addition to the series saturable reactor connection discussed in the previous section, a parallel winding arrangement is also in popular usage. Figure 10 shows the circuit connections for such a parallel arrangement. The explanation contained in this section will be based upon operation under natural magnetization conditions. In this case natural magnetization operation is characterized by a steady or smooth control current. After examination of the waveforms of figure 11 presented in conjunction with the parallel operation the reason for describing the operation as natural magnetization will become apparent.

In figure 11 the interval 0 to α finds the entire supply voltage developed across the gate windings. Accordingly the flux waveforms will increase. Since the flux in core A is biased by the d-c control current core A will saturate at angle α . No voltage will then be sustained over either gate winding since the two gate windings are in parallel. While not in saturation the flux of core B will be constant as a result of the zero gate voltage. With the supply voltage now developed across the load resistance the existence of load current is indicated. During the second half cycle of operation the results will be similar except that core B will now be the one that is saturated.

During the first saturation interval the path through gate

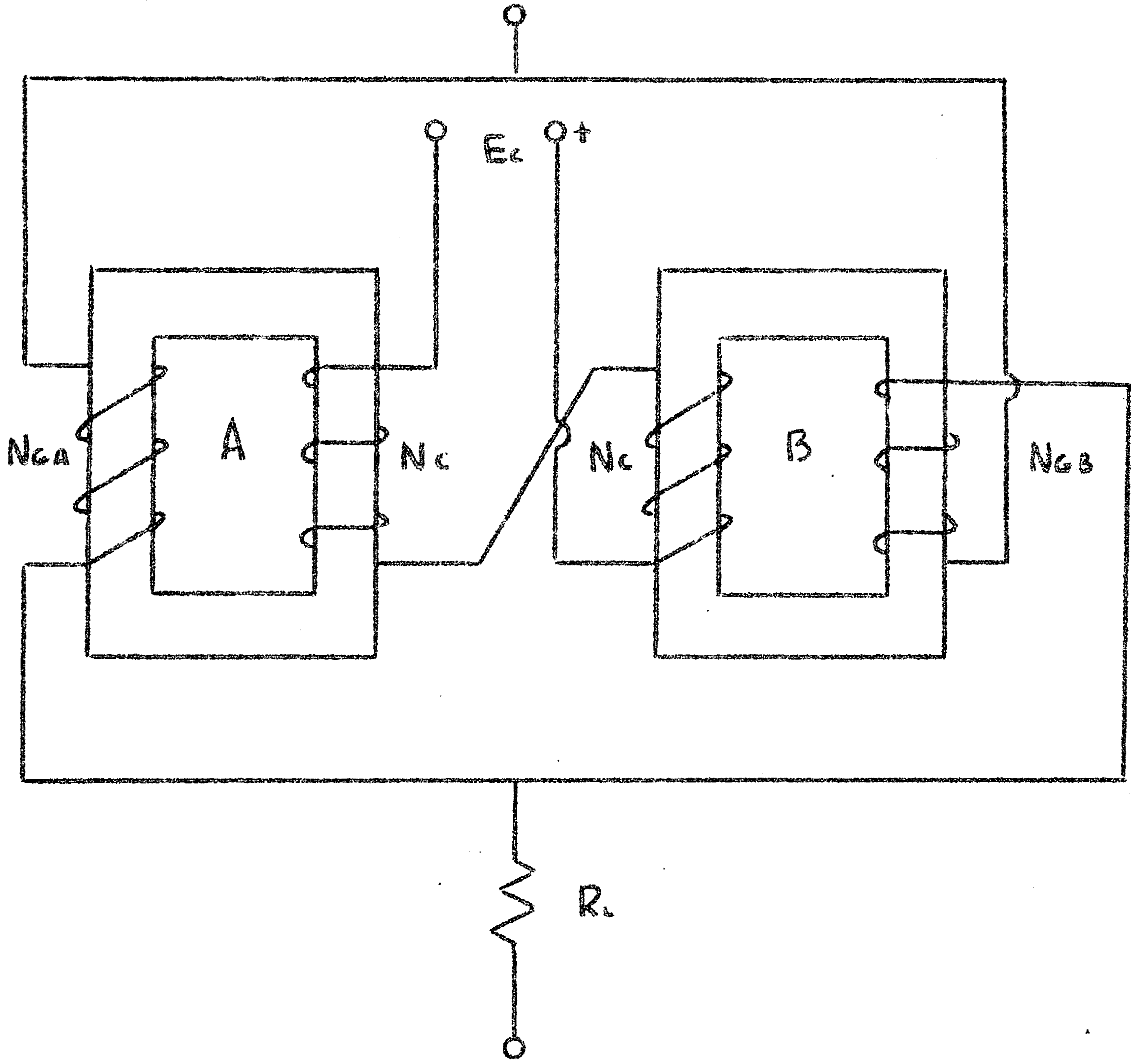

Figure 10 Parallel connected saturable reactor.

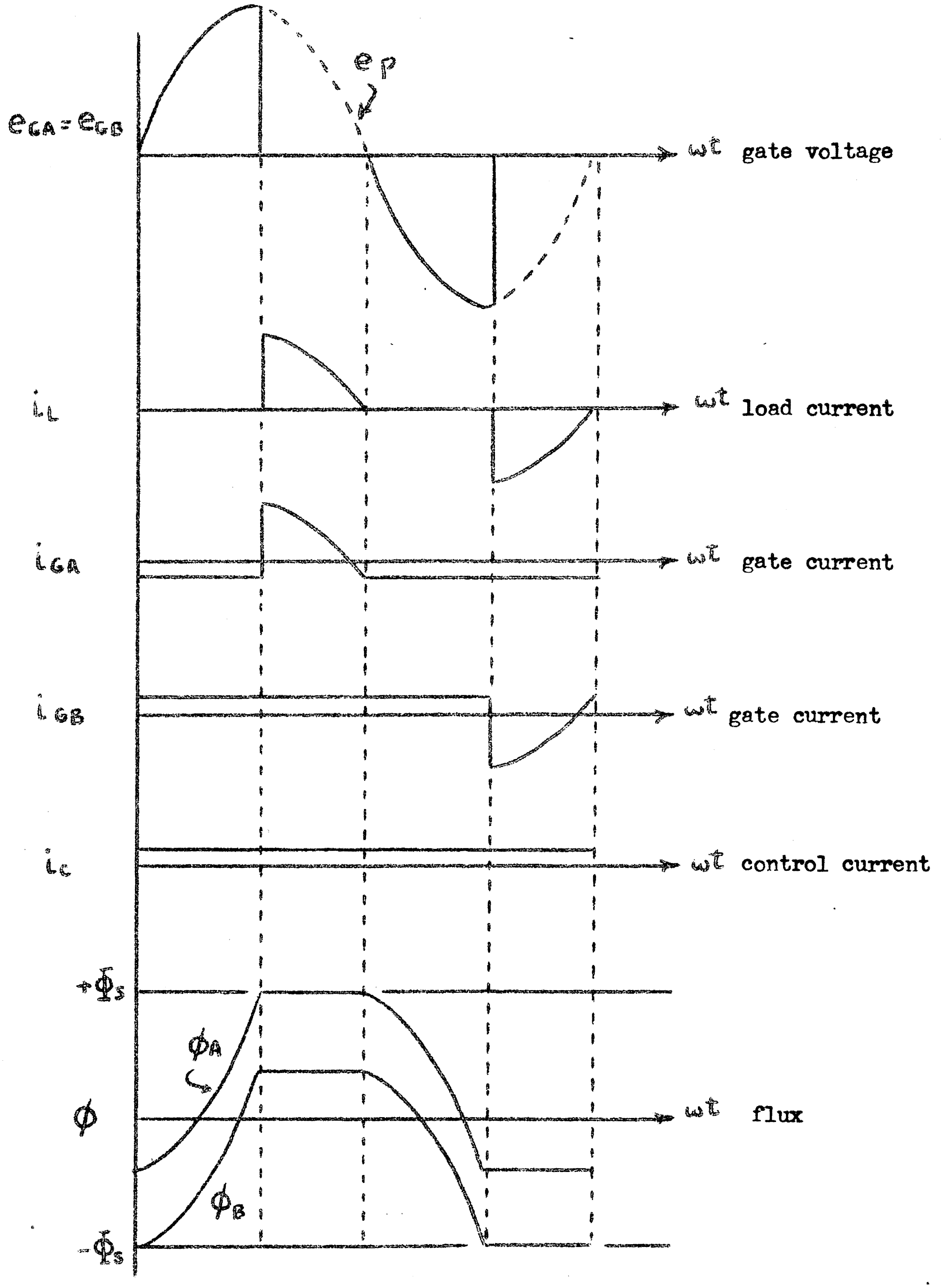

Figure 11 Voltage, current and flux waveforms for a parallel connected saturable reactor.

winding B will be a high impedance path due to the large referred control circuit impedance. On the other hand the impedance of path A will be low. The load current will be the gate current of A for the first saturation interval, and of B for the second. Since there is no d-c voltage source available in the gate and load circuit the gate current of each core must have a zero average value. Hence the offset current waveforms.

Notice that the free even harmonic currents which are characteristics of natural magnetization are found in the gate current components of the load current.

V. STEADY STATE SATURABLE REACTOR CHARACTERISTICS

Steady state operation of a saturable reactor is defined in terms of three variables. The supply voltage, the load current, and the control current are the three variables. The characteristic curves relating control current and load current with supply voltage as a parameter are commonly called the control characteristics. Supply voltage and load current with the control current as a parameter are related on the voltage-current characteristic curve.

Control Characteristic

The simple control characteristic can be evaluated from a series saturable reactor connection with the saturation voltage applied. Under these conditions the load current will be zero, or equal to the exciting current, when the control current is zero. Increases in the control current yield linear increases in the average load current. From the waveforms in figure 8 the increases in load current are characterized by a decrease in the firing angle α. The slope of the linear region, commonly called the proportional region, is described by the law of equal ampere turns. Continued increases in the control current will cease to yield increases in the load current when the firing angle reaches zero degrees. This point was described earlier as the maximum load current attainable. The only factors exerting any influence upon the load current at this point are supply voltage and load resistance. This region, for a

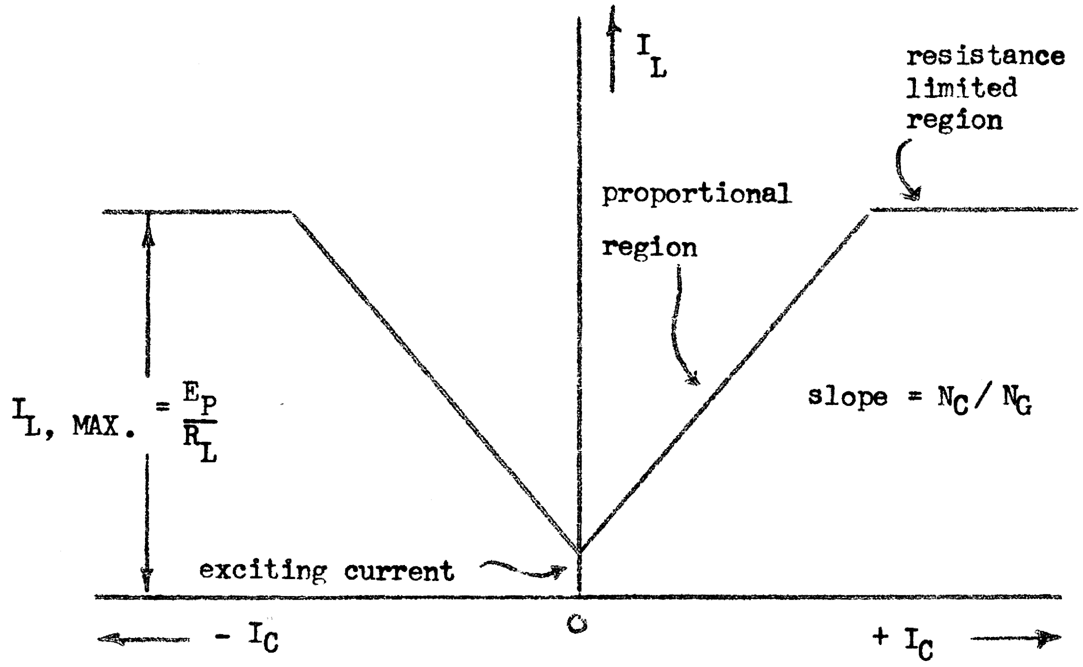

Figure 12 Simple control characteristic.

particular applied voltage, is referred to as the resistance limited region. The simple control characteristic is shown in figure 12.

Since the family of control characteristic curves may also depend upon the supply voltage three modes of operation are described. Normal excitation occurs when the supply voltage is equal to the saturation voltage. Underexcitation occurs when the supply voltage is less than the saturation voltage, and finally overexcitation occurs when the supply voltage is greater than the saturation voltage. The curves associated with the three modes of excitation are shown in figure 13.

In the underexcited case the control current must attain some value before it will succeed in biasing the flux curves sufficiently to produce saturation. This is shown in figure 13 by the flat portion of no output response to changes in control current. When saturation is reached, the linear curve will again exist with a limiting current value as before.

When the reactor is overexcited the control characteristic curve indicates that there is a minimum load current. Figure 7 indicates that without any control current, saturation occurs without the biasing of fluxes due to the supply voltage magnitude. This value of current is referred to as the irreducible minimum load current. Its average magnitude is defined as the difference between the saturation voltage and the supply voltage divided by the load resistance.

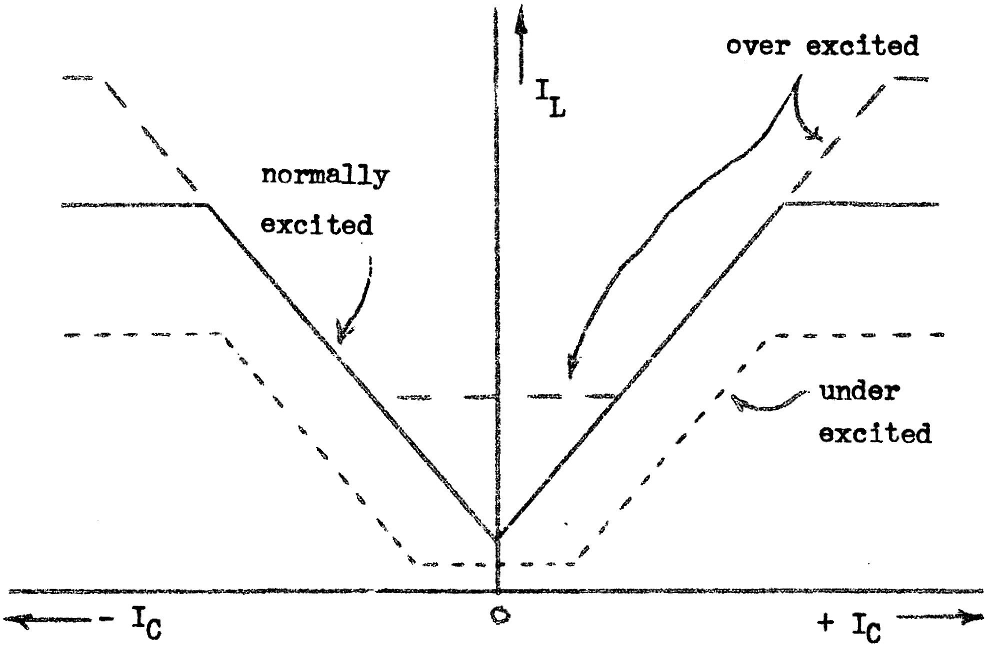

Figure 13 A family of control characteristic curves drawn for the three excitation modes.

Voltage-Current Characteristics

The voltage-current characteristics for a saturable reactor are normally plotted for a no-load condition. That is to say with R_L equal to zero. The characteristics for a particular load resistance can be derived from this by the application of a load line. A curve of voltage versus current is valuable in determining a satisfactory load resistance for the saturable reactor to work into.

Figure 14 (a) depicts the no-load voltage-current curve drawn to a scale of arbitrary units. For zero control current the only current in the load circuit will be exciting current as indicated until such time as the saturation voltage is reached. At this point the load current increases as the flux is driven more and more into saturation. For other values of control current the curves will be similar except that the vertical portions will be of a magnitude related to the control current by the ratio of turns.

The effect of the load resistance as expressed by the load line is a function of the drop across the load resistance due to the load current. The supply voltage with a load resistance in the circuit is the sum of the average no-load supply voltage (gate winding voltage) and the average load voltage.

The resulting curve of the characteristic response for a finite load resistance is drawn in figure 14 (b). By performing the operation described in the previous paragraph this construction is possible.

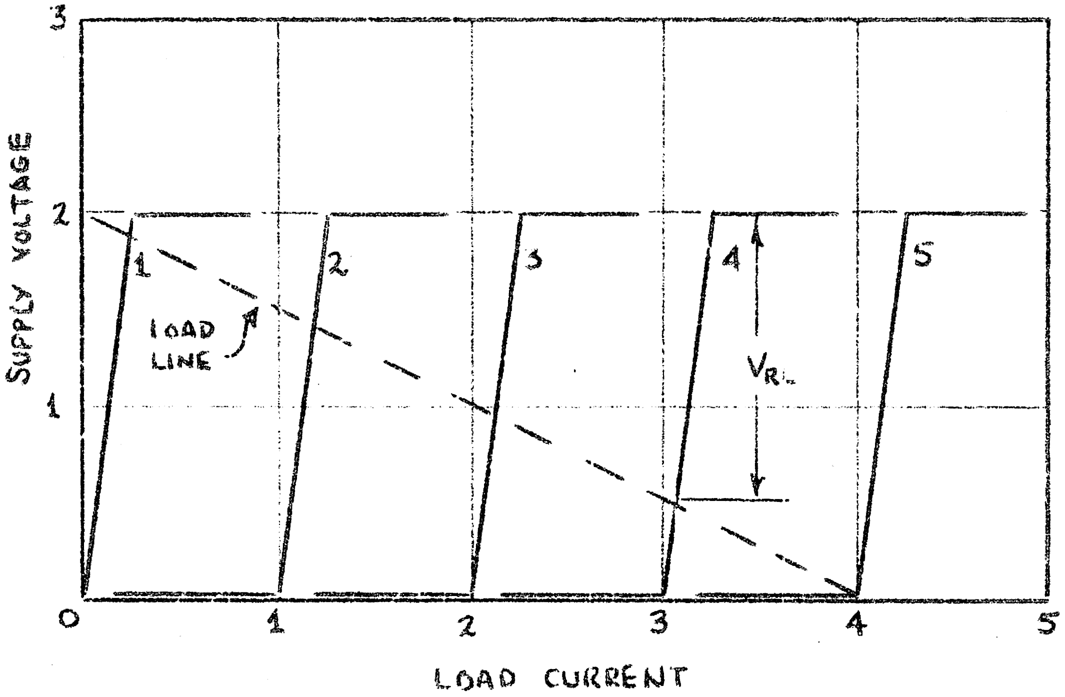

Figure 14(a) No-load voltage-current characteristic with load line.

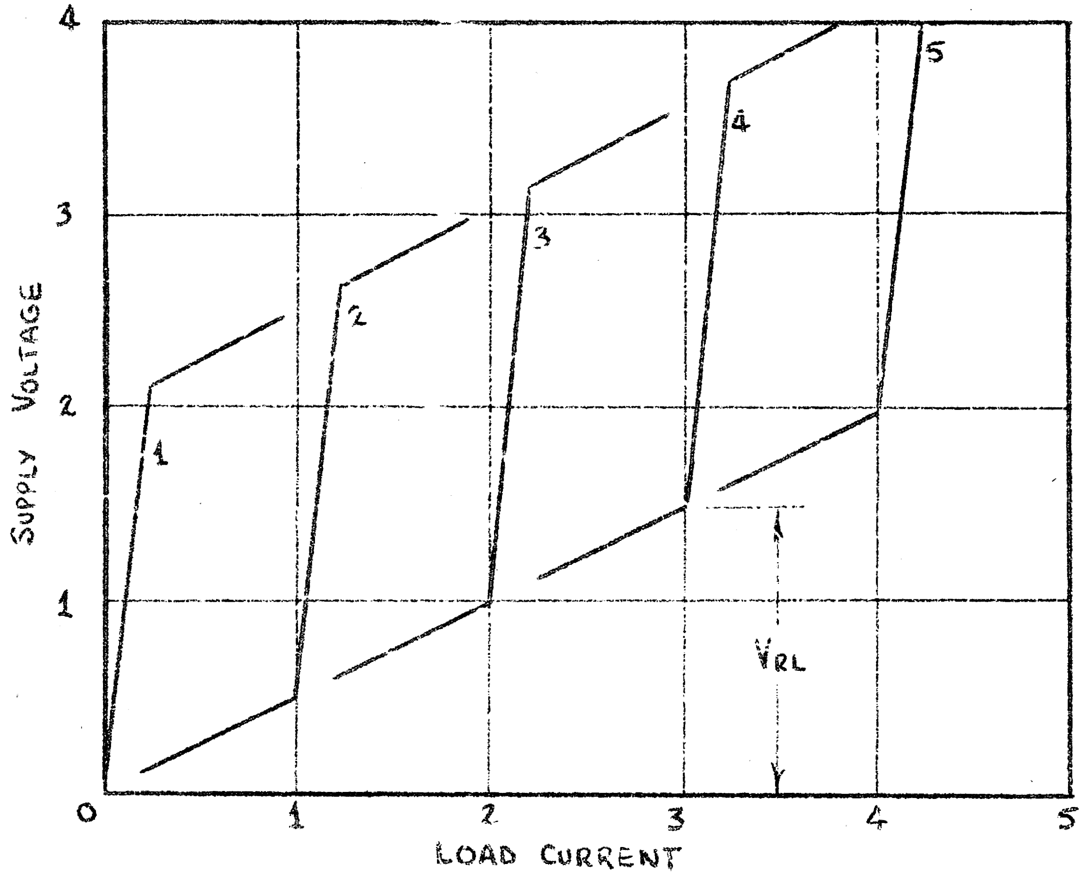

Figure 14(b) Voltage-current characteristic for load indicated in (a).

The selection of the proper load resistance in a saturable reactor circuit is found by referring to the family of voltage-current curves. The load line should be laid out so that changes in supply voltage for a specific value of control current do not cause an appreciable change in load current. The load line must also be laid out so that it covers the range of control currents to be used.

Power Gain

Power gain in a saturable reactor may be defined as the ratio of output power to input power. The output power is the power developed in the load resistance. The input power is that developed in the control circuit.[11] The power gain, K_p, of a saturable reactor without feedback may be shown by

$$K_P = P_{OUT}/P_{IN} = I_L^2 R_L / I_C^2 R_C = N_C^2 R_L / N_G^2 R_C \qquad 7$$

Current Gain

Over certain ranges of saturable reactor operation the load resistance and the supply voltage have no effect upon the current ratio I_L/I_C. It is on this statement that the definition of current gain is based.

$$K_I = I_L/I_C = N_C/N_G \qquad 8$$

The current gain could also be defined as the slope of the proportional region of the control characteristic. Under these circumstances the effect of feedback upon current gain will be more apparent.

VI. FEEDBACK IN MAGNETIC AMPLIFIERS

The saturable reactor as such, has a relatively low power gain limited by its design. Saturable reactors find many applications in this form, but additional applications would be available if the gain could be increased. The increase in gain may be accomplished through the use of feedback. By feedback is meant the utilization of a portion of the output quantity in such a manner so as to act for or against the input quantity. Naturally the increase in power gain comes about when the feedback assists the input. Feedback of this type is often called positive or regenerative feedback. Since feedback is a function of the output, namely load current, it may be achieved by several methods two of which will be described here; magnetic feedback and self-saturation.

Magnetic Feedback

Magnetic feedback is accomplished by using an additional set of windings on the two reactor cores. These windings are fed from the load circuit by a rectified form of the load current. Figure 15 shows a typical magnetic feedback arrangement. The feedback windings in figure 15 are connected in such a manner that their mmf will aid the control mmf.

The current gain of a reactor connected in this manner will be different from the current gain already mentioned by a factor involving the number of feedback windings N_F. The mmf equation

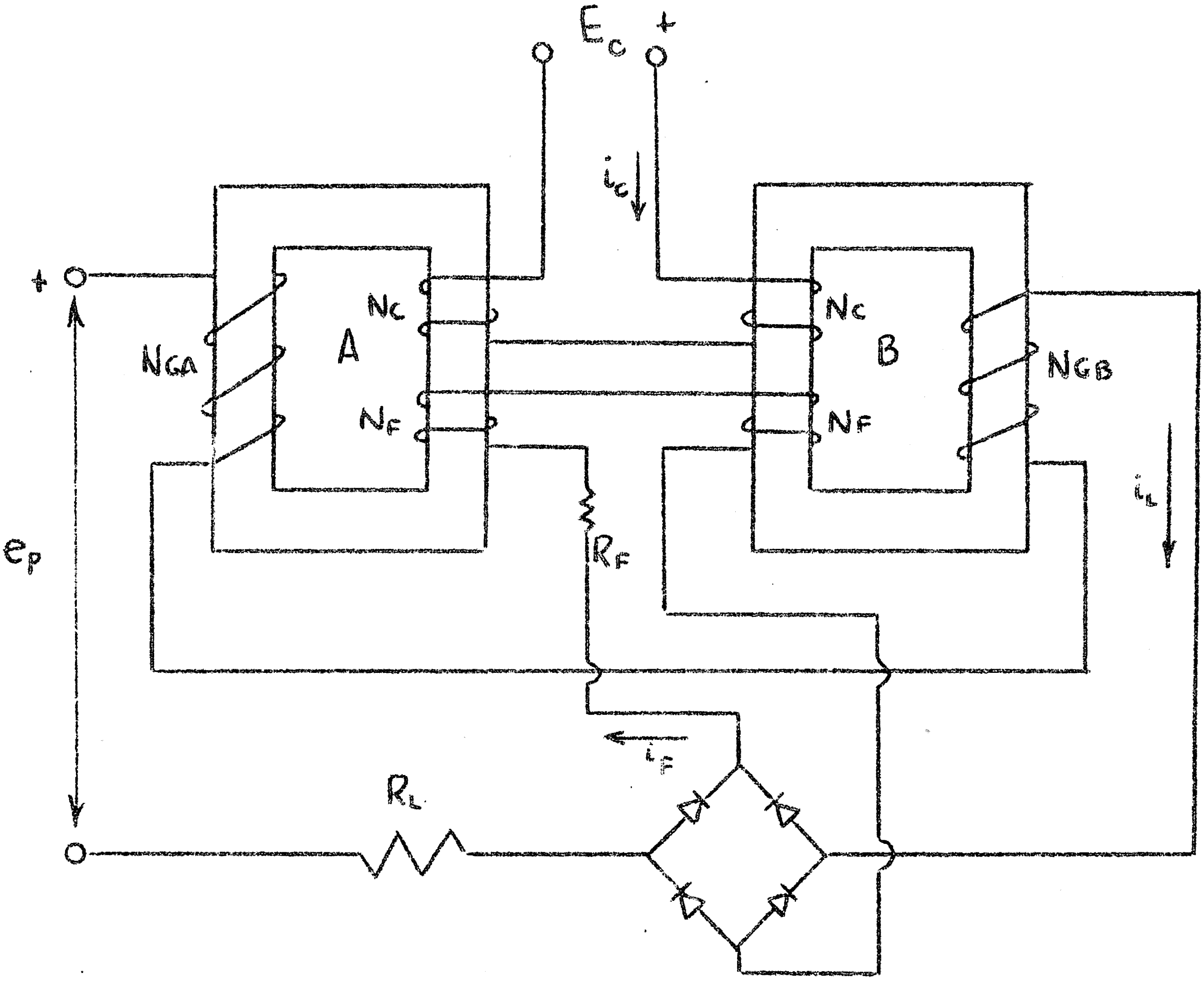

Figure 15 Magnetic amplifier, series connected, with magnetic feedback.

around one of the cores in figure 15 will be

$$I_c N_c + I_F N_F = I_L N_G \qquad 9$$

where I_F is the average of the current in the feedback winding. Solving equation 9 for the current gain yields

$$K_I = I_L / I_c = N_c/N_G \left[1 / 1 - N_F/N_G \right] \qquad 10$$

From the definition of load current and feedback current $I_F = I_L$, leaving the ratio of N_F/N_G in equation 10. The ratio of N_F/N_G is called the feedback factor. Notice that when $N_F = 0$ as is the case without magnetic feedback that the expression reduces to equation 8.

The addition of the feedback winding will alter the shape of the control characteristic by an amount dictated by the feedback current and feedback winding turns. This can best be shown graphically by redrawing the control characteristic in terms of control and load ampere-turns. This is done in figure 16. Since the feedback effect is additive to the control effect for the winding arrangement in figure 15, it may be plotted versus load current ampere-turns in the same manner as control ampere-turns. This curve of feedback ampere-turns is called the feedback line and is shown in figure 16.

The control characteristic for an amplifier with feedback can be constructed using the feedback line and an ordinary control characteristic. In figure 16 it may be seen that for a load current mmf of AB ampere-turns it is necessary to have a control mmf of CA ampere-turns without feedback. With feedback, though, the feedback

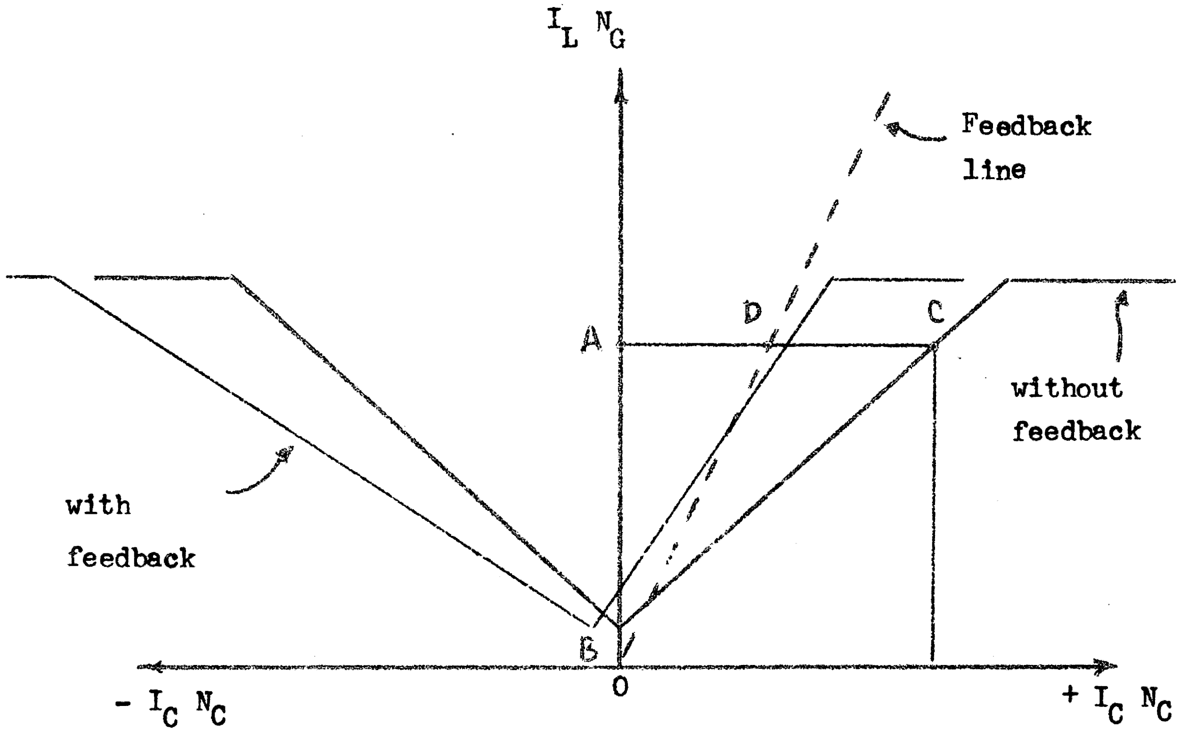

Figure 16 Relation between control characteristics with and without feedback.

current would produce AD ampere-turns at this same value of load current. This indicates that it is only necessary for the control winding to produce the difference, or DC ampere-turns. Plotting DC control mmf and AB load mmf a point on the new characteristic is obtained. Other points may be gotten the same way.

When the control current is increased in a positive direction the flux produced by the control windings aids that produced by the feedback windings. The core fluxes are then biased further apart resulting in a smaller firing angle. As a result of this the load current increases more per unit change in control current. This situation will continue until the firing angle reaches zero degrees. At this time the load current will be limited by load resistance as previously described.

If the control current is increased in the negative direction the fluxes produced by the control and feedback windings oppose. The firing angle will get larger for a period causing further reduction in load current. At the point where the control mmf and feedback mmf are equal and opposite the load current will be a minimum. Further increases in the control current produces a flux that overcomes the flux produced by the feedback winding. The result will again be a decrease in the firing angle and therefore an increase in the load current. One could say that the magnetic amplifier with feedback is polarized. That is to say that the reaction to positive and negative values of control current is different.

The effect of feedback on the power gain can be found from its definition and from the current gain expression of equation 10.

$$K_p = K_I^2 \, R_L/R_c = N_c^2/N_G^2 \, R_L/R_c \left[1/1 - N_F/N_G\right]^2 \qquad 11$$

Control Characteristic for Feedback Factors of Unity or Above

As the feedback factor is increased to unity and above the mode of operation is called infinite gain and snap action respectively. This type of operation is unstable due to the two operating points that exist for a single value of control current. Actually, in the interest of good stability, the feedback factor must be kept below 0.85.

A magnetic amplifier with infinite gain exists when the feedback factor is equal to unity. This means that the slope of the feedback line will be parallel and to the right of the proportional region of the control characteristic without feedback. The control characteristic for this situation is shown in figure 17.

A magnetic amplifier with snap action operates in an interesting manner. Referring to figure 18 the magnetic amplifier is operating at its quiescent point Q. A negative increase in the control voltage will move the operating point along the characteristic toward point A and finally to point B. If the control voltage is increased a slight bit more a reduction in core flux results along with a reduction in load current. This puts the operating point at point C where the applied control mmf is larger than that required for operation. The surplus control voltage is developed across the control winding further reducing the core flux and load current

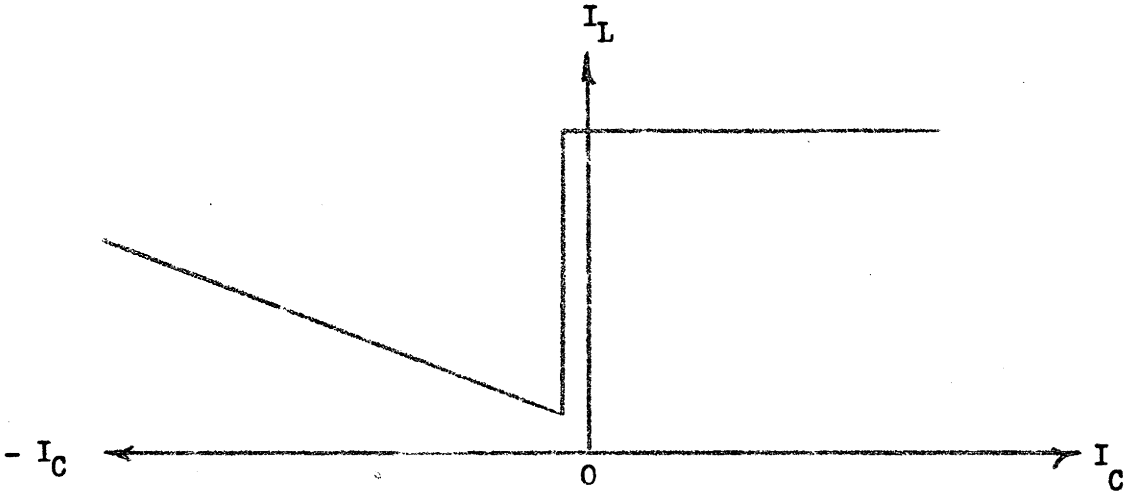

Figure 17 Magnetic amplifier with infinite gain.

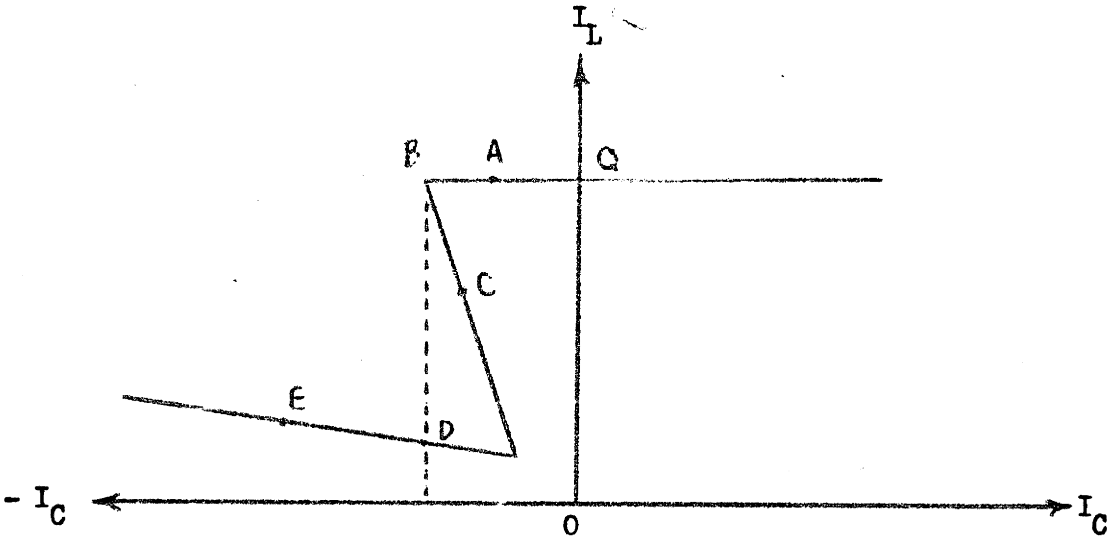

Figure 18 Magnetic amplifier with snap action.

until point D is reached. From this point on the operation is stable.

When the feedback factor is raised to a point greater than unity the result is a large change in load current for a minute change in control current. In the case of relay operation the change in relay current from a small value below the relay pickup current to a large value above the pickup value produces a rapid sure contact action.

It should be emphasized that it takes a certain amount of time for the load current to snap from B to D in figure 18. The time during which these changes occur is short, but it is finite. Oscillograms taken during the snap period prove this point and also prove that the control and load currents are actually moving along the s-shaped characteristic.[12]

Feedback by Self-Saturation

Feedback may also be accomplished by causing a direct current to flow in the gate windings. This is done by inserting series rectifiers into the load circuit. If the mmf produced by this direct current aids the control mmf then it is classed as positive feedback.

In its simplest form self saturation may be shown in terms of a basic, single core circuit shown in figure 19. The only difference between this circuit and the original basic saturable reactor circuit is the series rectifier in the load circuit.

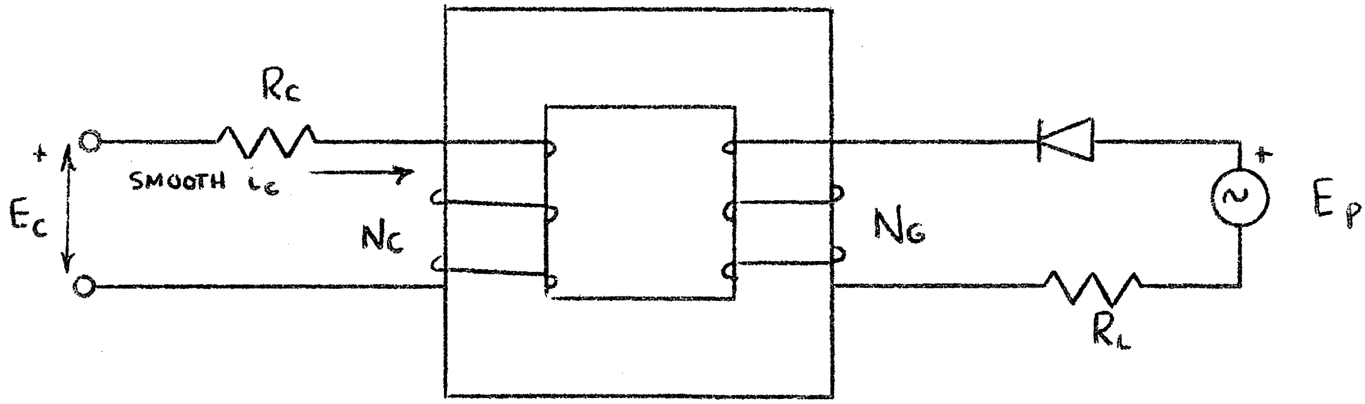

Figure 19 Basic single core self-saturating magnetic amplifier.

The steady state operation of the self-saturating device is described in the waveforms of figure 20. These waveforms are based on the assumption that the control current is constant. Between $wt = 0$ and $\propto$ the applied voltage is developed across the gate winding due to its high reactance. The flux variation during this interval is determined by the gate winding voltage. The current that exists in the load circuit will be the core exciting current.

At $wt = \propto$ the flux has been driven into saturation and the entire supply voltage will developed across the load resistance. This statement is true if the gate winding resistance and rectifier forward resistance is negligible. The load current has the form shown in figure 20 (c). At $wt = \pi$ the core becomes unsaturated reverting the gate winding back to its high inductance state. The load current continues to flow trailing off at $wt = \propto + \pi$. During this interval the rectifier continues to conduct the trailing current and the voltage across the rectifier will be negligible. The gate winding voltage exists so that the flux may reset itself to its original value Φ_1.[13] At $wt = (\propto + \pi)$ the rectifier assumes an infinite resistance and all of the supply voltage is developed across it.

Self-Saturation in Two Core Magnetic Amplifiers

Possibly the main point of self-saturation is the unidirectional current which exists in the gate winding. The direct current

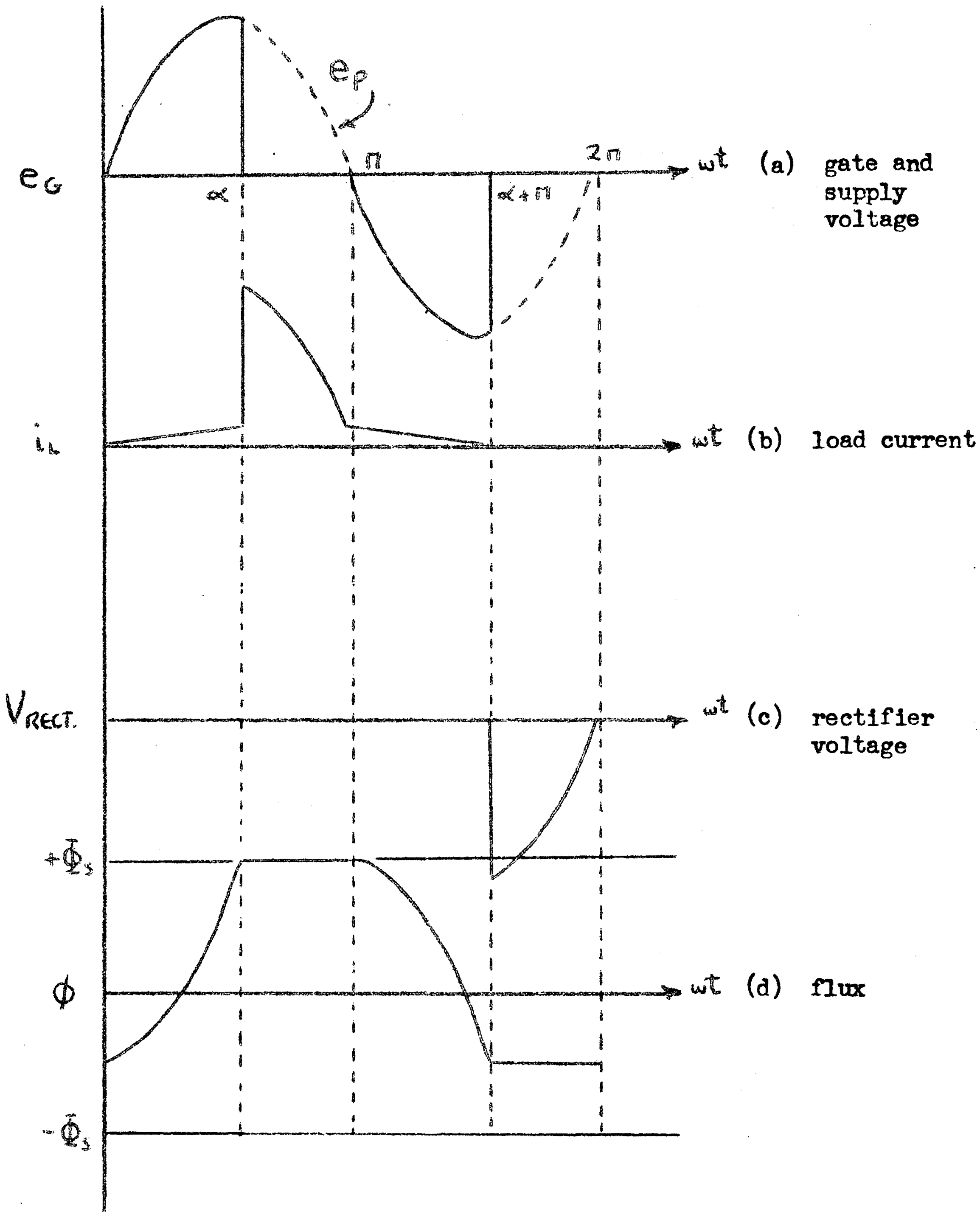

Figure 20 Waveforms depicting the operation of a single core self-saturating device.

component of this pulsating current acts to produce the feedback effect. Figure 21 shows the circuit arrangement of a magnetic amplifier employing self-saturation.

Consider the supply voltage polarity shown in figure 21. Current would exist in the gate winding of core B because of the arrangement of rectifier B. During the subsequent half cycle rectifier A will conduct, and a pulse of current will exist in gate winding A. In each gate winding there appears a unidirectional pulse of current. This situation will yield the necessary d-c component required for self-saturation. The result at the load is an alternating current fed first by one core and then the other.

The magnitude of the load current is dependent upon the magnitude and polarity of the control voltage. With a d-c polarity such that the flux produced by the control voltage aids that produced by the gate winding the result will be a higher magnitude of load current. Increases in the control current will drive the firing angle to zero and the load current will be limited by the circuit resistance.

If the polarity is reversed in the control circuit the fluxes produced by the two windings will be in opposition. The result will be a lowering of the load current level. This will continue until the fluxes are equal and opposite. This cancelling of fluxes is represented by the minimum current point. Further increases in control current will produce an over-riding flux which

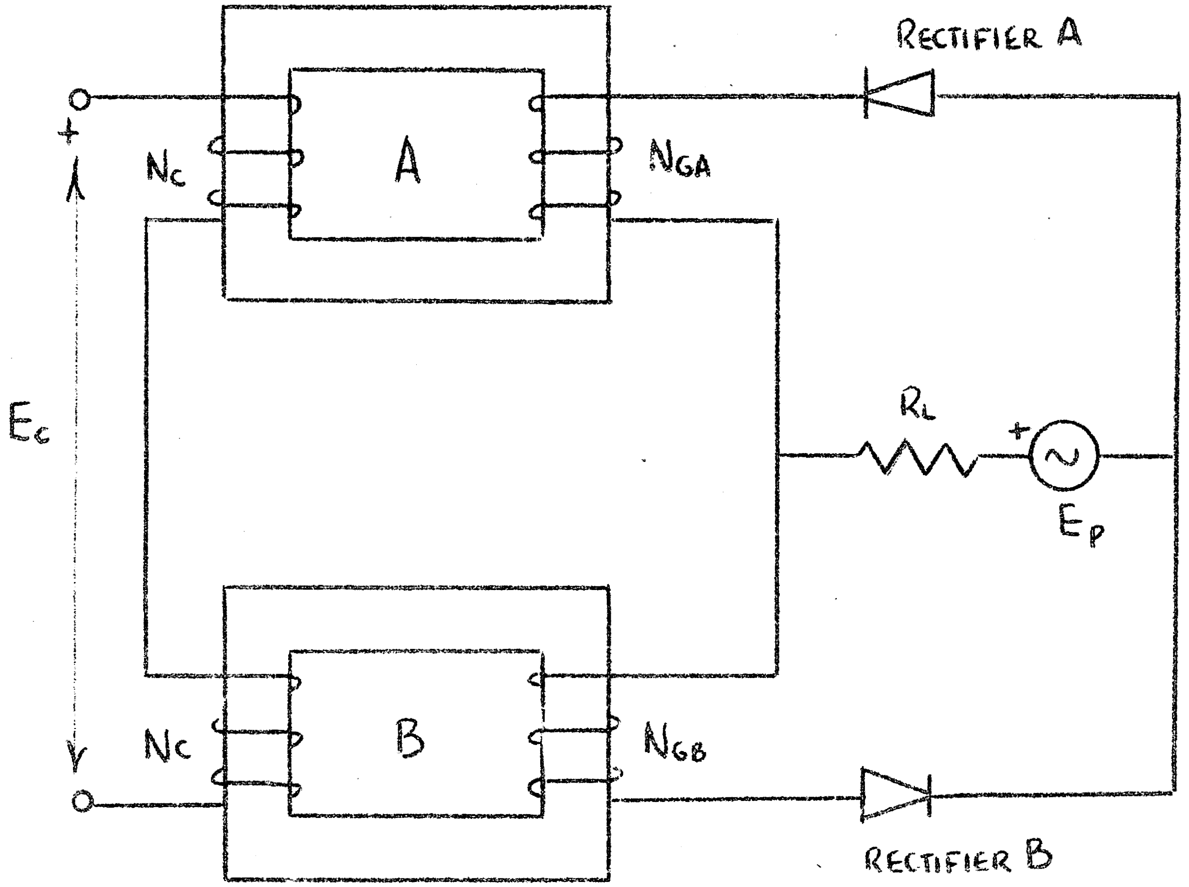

Figure 21 Magnetic amplifier with self-saturation.

will result in a slow increase in load current. From this description one can see that the control current versus load current characteristic curve exhibits the same qualities as in the case of magnetic feedback.

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AN INTRODUCTION TO THE STUDY OF MAGNETIC AMPLIFIERS

by

John P. Gordon
Virginia Polytechnic Institute
Department of Electrical Engineering

MASTER OF SCIENCE THESIS

June, 1959

ABSTRACT

The magnetic amplifier is a device employing saturable reactors as its basic element. The theory of magnetic amplifiers is developed from the standpoint of rectangular hysteresis loops that are simplified in such a manner as to eliminate the problems of nonlinearity.

The saturable reactor may be connected in either series or parallel combinations and their operation analyzed on the basis of voltage, current, and flux waveforms.

The characteristics of the saturable reactor circuits may be enhanced by the addition of feedback. Feedback is accomplished either by magnetic means using additional windings, or by self-saturation in which rectifiers are used to secure feedback. The result is a device in which a small amount of power can effectively be used to control larger amounts of power.